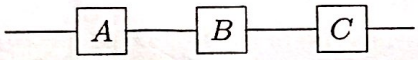
PART I: Multiple choice questions (4 points each with no penalty)

- Let Y be a Binomial random variable with parameters n and p. Find p if $E(Y) = 3Var(Y)$

$E(Y)=np$
$Var(Y)=np(1-p)$
$np = 3np(1-p)$
$\frac{1}{3} = 1-p$
$p = 1-\frac{1}{3} = \frac{2}{3}$

a) $\frac{1}{3}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

- An electrical system consists of three components connected is series, as shown in the diagram



The system fails if at least one of the components fail. If each component has probability p of failing, what must the value of p be so that the system functions with probability 0.9 ?

a) 0.027 b) 0.126 c) 0.014 d) 0.035

$P(F) = p$ $P(\text{System functions}) = 1 - P(F)$

- Let A, B and C be three independent events with respective probabilities $1/2, 1/3$ and $1/4$. Find $P(A \cup B \cup C)$

a) 1/24 b) 3/4 c) 1/4 d) 5/24

- Let X be an exponential random variable with pdf

$$f(x) = 0.4\, e^{-0.4x} \ , \ 0 < x < \infty$$

Determine the 40^{th} percentile

a) 0.204 b) 0.3 c) 2.29 d) 1.28

- If there are 7 people in a room, what is the probability that no two of them celebrate their birthd during the same month ? Assume that a person is equally likely to be born in any month, and t the group is randomly selected

a) 0.8886 b) 0.5833 c) 0.1114 d) 0.4167

- The median lifetime of a certain type of batteries is 2 years. Find the probability that exactly batteries last more than 2 years in a set of 3 such batteries

a) $\frac{1}{8}$ b) $\frac{2}{8}$ c) $\frac{3}{8}$ d) $\frac{4}{8}$

- The following measures were obtained for a sample of 25 observations:

$$\sum x = 399 \ , \ \sum x^2 = 22981$$

If each observation is multiplied by -2, find the standard deviation of the modified data set

a) 52.62 b) -52.62 c) 105.24 d) -105.24

- Let X be a geometric random variable with pdf

$$f(x) = (2/3)^{x-1} \cdot (1/3) \ , \ x = 1, 2, 3, \cdots$$

Find $P(X \geq 8)$

a) $(2/3)^8$ b) $(1/3)(2/3)^8$ c) $(1/3)(2/3)^7$ d) $(2/3)^7$

- A password is to consist of three A's, three B's and three C's. How many possible passwords are possible if no two Cs are adjacent ?

a) 35 b) 700 c) 800 d) 200

- An IT department is monitoring the performance of a computer server starting on the beginning of May. If the probability that the server crashes on any given day is 0.1, find the probability that the second crash occurs on the twelfth day of the month

a) 0.191 b) 0.041 c) 0.038 d) 0.23

- Given the data set 3.5, 7.0, 8.2, 9.8, 19.3, 50.6. What is the maximum amount we can add to the smallest value without affecting the median ?

a) 6.3 b) 5.5 c) 4.7 d) 5.3

- The repair time (in hours) for a certain machine is a random variable with pdf

$$f(x) = 16xe^{-4x} \ , \ 0 < x < \infty$$

Find the mean repair time

a) 1.25 b) 0.5 c) 1.5 d) 0.75

- Let X be a Gamma-distributed random variable with mean 28 and variance 112. Find the probability that X lies between 12 and 24

a) 0.360 b) 0.420 c) 0.630 d) 0.240

- The grades of students in statistics class have a mean of 56 and standard deviation 3. At most what percentage of students have more than 63.5 ? Assume the distribution of the grades is symmetric

a) 0.08 b) 0.16 c) 0.84 d) 0.92

- Approximate the following sum by using Poisson: $\sum_{x=2}^{5} \binom{60}{x} (0.05)^x (0.95)^{60-x}$

a) 0.717 b) 0.817 c) 0.493 d) 0.847

PART II. Show your detailed work to receive full credit

- (8 pt.) The probability an electric motor is defective is 0.04. Find the probability that a sample of 300 motors will contain between 5 and 9 (inclusive) defective motors

$p(D) = 0.04$ X = no. of defective motors.

$P(5 \le X \le 9) = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9)$

$P(X=5) = \binom{300}{5} (0.04)^5 (0.96)^{295} = 0.0118$

$P(X=6) = \binom{300}{6} (0.04)^6 (0.96)^{294} = 0.0242$

$P(X=7) = \binom{300}{7} (0.04)^7 (0.96)^{293} = 0.0423$

$P(X=8) = \binom{300}{8} (0.04)^8 (0.96)^{292} = 0.0646$

$P(X=9) = \binom{300}{9} (0.04)^9 (0.96)^{291} = 0.0873$

$P(5 \le X \le 9) = 0.2302$.

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- Let X a random variable with probability density function

$$f(x) = \frac{x^2}{9}, \quad 0 < x < 3$$

(5 pt.) Find the probability that X is larger than 1 given that $X \leq 2$

$$P = P(X>1 / X\leq 2) = \frac{P(X>1 \text{ y } X\leq 2)}{P(X\leq 2)} = \frac{P(1<X\leq 2)}{P(X\leq 2)}$$

$$P(1<X\leq 2) = \int_1^2 \frac{x^2}{9} = \frac{1}{9}\frac{x^3}{3}\Big]_1^2 = \frac{x^3}{27}\Big]_1^2 = \frac{8}{27} - \frac{1}{27} = \frac{7}{27}$$

$$P(X\leq 2) = \int_0^2 \frac{x^2}{9} = \frac{x^3}{27}\Big]_0^2 = \frac{8}{27}$$

$$P = \frac{\frac{7}{27}}{\frac{8}{27}} = \boxed{\frac{7}{8}}$$

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(5 pt.) Find $E[X(X-1)]$

$$E(X(X-1)) = E(X^2 - X) = E(X^2) - E(X)$$

$$E(X^2) = \int_0^3 \frac{x^4}{9} = \frac{x^5}{5\times 9}\Big]_0^3 = \approx 5.4$$

$$E(X) = \int_0^3 \frac{x^3}{9} = \frac{x^4}{4\times 9}\Big]_0^3 = \approx 2.25$$

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$$E(X(X-1)) = 5.4 - 2.25 \approx \boxed{3.15}$$

(6 pt.) Find the cumulative distribution function $F(x)$

$$F(X) = \int_0^x f(y)dy = \int_0^x \frac{y^2}{9} = \frac{y^3}{3\times 9}\Big]_0^x = \frac{x^3}{27}$$

3†

$X<0$?

$X>3$?

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- (8 pt.) Two companies A and B, drill wells in a rural area. Company A charges a fiat fee of $3500 to drill a well regardless of its depth. Company B charges 1000 plus 13 per foot to drill a well. The depth of wells drilled in this area have a normal distribution with a mean of 250 feet and a standard deviation of 40 feet. Find the probability that company B would charge more than company A to drill a well

Company A: 3500$ for any depth.

Company B: $1000 + 13X$ / X = depth in feet.

Normal distribution of $\mu = 250$ feet & $\sigma = 40$ feet.

for company B to charge more than company A.

then. $1000 + 13x > 3500$

$13x > 2500$

$\boxed{X > 192.3}$

$P(X > 192.3) = P\left(Z > \frac{192.3 - 250}{40}\right)$

$= P(Z > +1.44)$

$= 1 - P(Z < 1.44)$

$= 1 - 0.9251$

$= \boxed{0.0749}$

are of type B. A box is chosen at random then 3 processors are selected at random and without replacement from this box.

(4 pt.) Find the probability that one processor is defective among the three selected

D: 1 processor is deffective among the three selected

A: defective is from A

B: defective is from B.

$$P(D) = P(A \cap D) + P(B \cap D)$$

$$= \frac{\binom{2}{1}\binom{8}{2}}{\binom{10}{3}}(0.4) + \frac{\binom{1}{1}\binom{9}{2}}{\binom{10}{3}}(0.6).$$

$$= \frac{14}{75} + \frac{9}{50} = \frac{11}{30}$$

(4 pt.) Given that one processor is defective among the 3, what is the probability that it is the only defective processor in the box ?

O: only defective processor in box

$O = B$

$$P(B/D) = \frac{P(B \cap D)}{P(D)}$$

$$= \frac{P(D/B)P(B)}{P(D/B)P(B) + P(D/A)P(A)} = \frac{\frac{9}{50}}{\frac{11}{30}} = \frac{27}{55} = 0.4909$$